CHAPTER 2. BALLISTICS IN ONE DIMENSION

In this section we consider the motion of projectiles traveling straight up and down, and of vehicles and other objects going horizontally. In many cases of interest the motion involves constant acceleration, the most common example being the acceleration of gravity close to the Earth.

2.1 Position, Velocity, and Acceleration

Position is just location in space, usually on some grid like Cartesian coordinates. It is typically measured in feet, or meters.

Velocity is the rate of change of position of an object. It is a vector, with a magnitude, equal to the **speed,** and a direction. It is typically measured in feet per second, or meters per second.

Acceleration is what you do when you make an object go faster or slower. It's the rate of change of the velocity. In this course, we always assume that the acceleration is constant. In this way, we can do a lot of important problems without calculus. Acceleration is measured in feet per second squared, or meters per second squared m/s^2.

It s important to remember that velocity is a vector it has a direction and a size. Speed has no direction just a size. This, at least, is how physicists define these terms.

In one dimension, especially with constant acceleration only, life is easy. Choose a coordinate, say *x*. This is not to be confused with a distance--it's a position on a Cartesian axis, and can be positive or negative. Measure the position of a particle twice during its motion, at different times, and an **average velocity**, $\mathbf{v}_{\mathbf{av}}$,can be defined as follows.

This looks just like the definition of the slope of a line in the *x*-*t* plane, and it is--that of a line that goes through two points of the trajectory. To get the exact velocity at a single point, you have to make very small–slide the second point backwards until it's on top of the first point. This is called the **instantaneous velocity**.

An **average acceleration** can also be defined:

An instantaneous acceleration can be defined, again by making extremely small.
In this course the size of the acceleration is always taken to be constant, the same at all times.

2.2 One-Dimensional Ballistics

Suppose you have an object moving at constant acceleration, along with the velocity and position at some particular time. How do you find the velocity and position at any subsequent time?

You do it by plugging into one or more of the following equations:

What do all these symbols mean?

v = velocity
a = acceleration
t = time
v_0 = starting velocity--usually when $t=0$.
x_0 = starting position

With a little algebra, it s possible to get a third equation from these two:

When acceleration is constant, as for the gravity field close to the Earth, the motion can be graphed as a parabola in the x-t plane. The velocity, meanwhile, is a straight line. See the figures, for a ballistic projectile launched straight upwards from a height, like a cannonball fired from a cannon on top of a cliff.

The above three equations are always true for constant acceleration in one dimension. Along with the ideas of acceleration and velocity, they are the most important equations in one dimensional ballistics. To solve all these constant acceleration problems, use the following method.

General Method, 1-dimensional Ballistics

Step 1. Write down the ballistic equations.
Step 2. Reread the problem and identify all the constants.
Step 3. Plug the constants in, and solve for the unknowns. Most problems reduce to two equations and two unknowns.

It's not a bad idea to make a short table of all quantities in the problem, and fill in what you know. What you don't know, you usually have to find out.

2.3 Relative Velocity in One Dimension

In one dimension this is a very easy problem, usually involving subtraction of one velocity from another. SIGNS ARE IMPORTANT, of course. To be systematic, it's best to use the following formula:

This formula reads: "the velocity of A relative C is equal to the velocity of A relative B plus the velocity of B relative C.

Technique: Relative Velocity

Step 1. Identify A, B, C. One of them is almost always the Earth. The other two could be, say, a train and a car. By the way, it's best to use letters that remind you what the thing is, for example E for Earth, T for train, and C for car.

Step2. Write down the three velocities. Usually only two of them are known, and the other must be found.

Step 3. Two of these velocities will have a common letter which appears upstairs in one and downstairs in the other. Put these two on one side of the equation, the other on the opposite side.

Step 4. Solve for the unknown velocity.

2.4 Examples

2.4.1 Position, Velocity, and Acceleration

Example 1: A car accelerates from rest to 40 meters per second in 8 seconds. Find (A) the average acceleration. (B) the average velocity, if the car covers 200 meters in this time.
Solution: This is straightforward.
(A)

(B)

2.42 One-Dimensional Ballistics

Example 2. A boy throws a bowling ball straight up into the air with an initial speed of 5 m/s. (A) How long does it take to get to its maximum height, where the velocity is equal to zero? (B) What is the maximum height above its release point? Use for convenience.

Solution: Apply simple ballistics. (A) At the maximum height, v=0, so

(B) To find the maximum height, plug into the height equation. Assume the initial height is zero, which is the height at which the ball leaves the boy's hand.

Example 3. Fred Flintstone, traveling in his prehistoric car initially at 20 m/s, brakes uniformly with his foot. If it takes him five seconds to come to a stop (A) what is his acceleration? (B) Through what distance did he travel?

Solution: Again, simple ballistics, but horizontally, this time. Notice that this is really almost identical to the previous example . (A) We know that 5 seconds are necessary for the velocity to drop to zero. We can use this to find the acceleration. 'Uniformly', by the way, is another way physicists have of saying 'constant'.

(B) Apply the position formula.

Example 4. A grasshopper jumps straight up, barely making it to the branch of a bush 0.8 meters off the ground. (A) What was his initial velocity? (B) How long did it take to reach his maximum height?

Solution: Here, it's handy to use the third ballistics formula, involving the squares of the velocities. This is true whenever velocities and positions are given. (A) Plug in. We're looking for v_0.

(B) Now we're back to the position equation. We'll need the quadratic formula. Set the height equation equal to the maximum height, and solve for t.

You might get some truncation error, and thus something slightly different from 0 inside the square root, but this problem does, indeed, have a double root. This can be guessed by the symmetry--there can be only one time at which the maximum height is achieved, whereas other heights have two answers--one for going up, and a second for going down.

Example 5. A rocket car accelerates from rest at a constant rate of 5 m/s^2. (A) How long does it take to reach 400 m/s? (B) How far has it traveled in that time?
Solution: Use the ballistics equations.
(A)

(B) Now use the position equation:

Let the starting point be . (Generally, this is user selectable). Since , we have

Example 6. An airplane comes in for a landing, hitting the tarmac at 100 m/s, decelerating at a constant rate. If it comes to a stop within 500 meters, what was the acceleration?

Solution: This problem can be solved with the first two ballistics equations, but it's easier to use the third.

Example 7. (A) Suppose a ball is thrown straight up into the air. Given that and an initial velocity of , and the initial position is 30 meters, find the velocity and position functions. (B) How high does the ball go? (C) How long does it take to reach half its maximum height?

Solution: (A) This is a simple matter of plugging the initial conditions into the ballistics equations.

These are simple ballistics equations describing the height of a particle in a constant gravity field. The usual acceleration of gravity at the surface of the Earth is about , but ten gives reasonable results when high accuracy isn't essential.

(B) To find the maximum height, plug in v=0 and solve for t:

Put this time into the height equation, getting

(C) To find how long it takes to reach half its maximum height, simply set the height function equal to 25 meters.

There are two answers, since half-height is hit on the way up and on the way down.

Example 8: Supergirl. A lady, caught by a gust of wind, falls off the torch of the statue of Liberty. Supergirl, hanging out at a height of two kilometers, catches sight of her when she is 40 meters off the ground and traveling at terminal velocity of 50 m/s. Terminal velocity means the woman remains at constant speed, due to air drag. Starting from rest, what must supergirl's net acceleration be, if she is to break the woman's fall in the nick of time?

Solution: First, find out the maximum time the woman will remain in the air after Supergirl spots her. The woman's net acceleration is zero, so

Supergirl has 0.8 seconds to get to the woman. Starting from rest at a height of 2,000 meters:

A little algebra gives

Neither gravity nor the air friction on Supergirl had to be figured in separately--it's built into the final answer. \

Example 9. Moon Jump. Suppose you can jump one meter high on Earth.
How high could you jump on the moon, where the acceleration of gravity is one-sixth that of Earth? Neglect the fact you'd have to wear a spacesuit on the moon.

Solution: Assume that your legs can deliver the same initial velocity on the moon as on Earth. This would not necessarily be the case, since gravity is decelerating the body even while in the act of pushing off from the ground, but the error would be small. First, we find the intial velocity on the Earth. At the maximum height, the velocity is zero, so

Now plug this into the third ballistics equation.

Example 10. High Speed Chase. A man in a Mercedes moves at constant
velocity of 40 m/s. A cop watches him pass by, then finishes eating his donut, which takes ten seconds. The policeman then takes off after the Mercedes, accelerating at 2 m/s^2. How long

does it take him to catch up with the Mercedes?

Solution: This is a two-variable problem, a warmup for the next chapter. In this case, both variables represent positions in the same direction. When the cop starts to give chase, the car has traveled 40 meters. So

We can find out how much time is needed for the policeman to catch the car by setting . This gives

Take the positive root, of course, because interception occurs after the chase begins, not before!

The cop, at this point will be traveling nearly 100 m/s (360 km/hr!) and will have to brake heavily.

Relative Velocity

Example 11: Train vs Car. A train is going north at 50 m/s. A car is going south at 30 m/s. What is the velocity of the train relative the car?

Solution: The train is going 50 m/s with respect to the Earth. Designate that

. The car is going south at 30 m/s with respect to the Earth. Designate that velocity . We're looking for the velocity of the train relative the car, which is . Arrange these three quantities in an equation, with the two velocities having the same letter upstairs in one and downstairs in the other on the right hand side:

Solve for and plug in the values.

This is the intuitively obvious answer, but when we go to two dimensions, the technique developed here will make life a lot easier.

Example 12: Helicopter vs. Airplane. Suppose a helicopter and an airplane are separating at 60 m/s. If the helicopter is flying west at 25 m/s, at what velocity is the airplane flying, going east?

Solution: This problem is similar to the previous, of course. Choose west to be negative, east to be positive. We are given that , the velocity of the airplane relative the helicopter. We are also given that , the velocity of the helicopter relative the Earth. We want to find , the velocity of the airplane relative the Earth. "H" appears both upstairs and downstairs in two of the terms, so these two terms must appear on the right hand side, together:

Example 13. Police Relative Car. A car going 90 mph passes a policeman at rest. The policeman gives chase, at 120 mph. The car, a lotus, accelerates to 150 mph. What is the velocity of the policeman relative the lotus?

Solution: The point of this exercise is to show that a relative velocity can, of course, work out to be negative. We have: