Modern Physics Test 2 Review

November 15, 2001

1. Solve Schrodinger's equation for a particle trapped in a two-dimensional infinite potential well with $0 < x < L$ and $0 < y < L$.

Solution: see Rohlf.

2. A proton of energy 200 eV is trapped inside a potential well given by

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \\ 0 & \text{if } 0 < x < 5\ fm \\ 1000 - 100x & 5 < x < \infty \end{cases}$$

Calculate the probability that the particle escapes the well, using the approximate method.

Solution: Use the tunneling probability estimate developed in class notes.

$$P = \exp\left(-2\int_{x_1}^{x_2} \sqrt{2m(V(x) - E)/\hbar}\right)$$

where x_1 and x_2 are the turning points, that is, those points for which either the potential terminates (as here), or where $V - E = 0$.

3. (A) Calculate the wavelength of the photon emitted by an electron in the hydrogen atom dropping from n=5 to n=2. (B) Same problem for Li^{+2}. (C) same problem, for muonic hydrogen.

Solution: (A) This is easy. Use

$$E_n = \frac{-13.6\ eV}{n^2}$$

and get

$$\hbar\nu = \hbar\frac{c}{\lambda} = E_5 - E_2$$

(B) Same act, but different Z. This is a hydrogen-like atom.

$$E_n = -\frac{13.6Z^2}{n^2}$$

(C) Same act, but different m. Use the reduced mass,

$$\mu = \frac{mM}{m+M}$$

in a Bohr model to get the new energy expression. 4. Write down all possible states of a 3d electron (i.e. give the quantum numbers).

Solution: Ho hum.

5. What is the probability of finding an electron in the 2s state inside the Bohr radius?

Solution: Use the 2s wave function. In this case, we can't hack it as in the problems where the electron is found inside the nucleus, since the exponentials aren't approximately equal to one in the region under consideration.

6. Calculate the different energy levels for 3d electrons in a 5 Tesla magnetic field.

Solution: This is a matter of plugging in.

$$E = E_n + \frac{e\hbar}{2m}\left(m_l + 2m_s\right) B$$

7. An electron is trapped in a finite symmetric potential well with V=1000 eV. The well is one angstrom wide. Find the lowest energy level.

Solution: For this one, use the boundary condition and the bisection method on the transcendental equation to find the lowest bound state. The equation is

$$\tan\left(\frac{L\sqrt{2mE}}{2\hbar}\right) = \sqrt{\frac{V_0 - E}{E}}$$

Plug in all the numbers, using electron volts. Notice that L is half an angstrom (because of the setup in Rohlf, -L/2 to +L/2), and that

$$\frac{\hbar^2}{2mL^2}$$

has units of energy. One trap: make sure your calculator is working in radians. We look for solutions of the transcendental equation

$$\tan\left(\frac{E}{60.93}\right) - \sqrt{\frac{1000}{E} - 1} = 0$$

The ground state, if I plugged in right, is about 78.4 eV. Notice the nice technique given in Rohlf for this kind of problem, where an initial estimate is obtained with the infinite well, which is used to get a better answer by successive iterations. This method depends on the ground state being much less than the height of the barrier, and uses the fact that the wave function is greatly attenuated within $1/\beta$ outside of the well (see 7.81).

8. A particle is trapped in an infinite well with $0 < x < L$. Find (A) the probability that the ground state particle is between 0 and L/3. (B) The normalization factor.

9. Given the following wave function in an infinite potential box, plug into Schrodinger's equation and determine the energy.

$$\psi = A\cos\left(\frac{3\pi x}{a_0}\right)$$

.where a_0 is some typical length scale, say the Bohr radius.